"Which-path information" and partial polarization in single-photon interference experiments

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Abstract

It is shown that the degree of polarization of light, generated by superposition in a single-photon interference experiment, may depend on the indistinguishability of the photon-paths.

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Quantum systems (quantons [1]) possess properties of both particles and waves. Bohr's correspondence principle [2] suggests that these two properties are mutually exclusive. In other words, depending on the experimental situation, a quanton will behave as a particle or as a wave. In the third volume of his famous lecture series [3], Feynman emphasized that this *wave-particle duality* may be understood from Young's two-pinhole type interference experiments [4]. In such an experiment, a quanton may arrive at the detector along two different paths. If one can determine which path the quanton traveled, then no interference fringe will be found (i.e., the quanton will show complete particle behavior). On the other hand, if one *cannot* obtain any information about the quanton's path, then interference fringes with unit visibility will be obtained (i.e., the quanton will show complete wave behavior), assuming that the intensities at the two pinholes are the same. In the intermediate case when one has partial "which-path information" (WPI), fringes with visibility smaller than unity are obtained, even if the intensities at the two pinholes are equal. For the sake of brevity, we will use the term "best circumstances" to refer to the situation when in an Young's interference experiment, the intensities at the two pinholes are equal, or to equivalent situations in other interferometric setups [5]. The relation between fringe visibility and WPI has been investigated in Refs. [7–9]. It has been established that a quantitative measure of WPI and fringe-visibility obey a certain inequality [10].

Aim of this paper is to show that it is not only the fringe-visibility, but also the polarization properties of the superposed light, which may depend on WPI in an interference experiment. In particular, it will be established that the degree of polarization and a measure of WPI obey an inequality, with equality holding under the "best circumstances".

Polarization properties of light have been extensively studied in the framework of classical theory (see, for example, [11–14]). Foundation of this subject was laid down by Stokes in

1852 in a classic paper [15] in which he formulated the theory of polarization in terms of certain measurable parameters, now known as Stokes parameters (see, for example, [12], p. 348). In the late 1920's, Wiener [16–18]) showed that correlation matrices can be used to analyze the polarization properties of light. Wolf [19] used the correlation matrix formulation for systematic studies of polarization properties of statistically stationary light beams within the framework of classical theory. Later, some publications addressed this problem by the use of quantum mechanical techniques (see, for example, [20–22]). A discussion on quantum mechanical analogues of the Stokes parameters can be found in Ref. [13], Appendix L. Recently, another quantum mechanical analysis of polarization properties of optical beams have been presented [23].

In this paper, we will mainly use the formulation of Ref. [23]. According to that formulation, the polarization properties (based on first-order correlation functions [24]) of light at a space-time point $(\mathbf{r}, t)$ may be characterized by the so-called quantum-polarization matrix

$$\overleftrightarrow{G}^{(1)}(\mathbf{r}, t; \mathbf{r}, t) \equiv \left[G_{ij}^{(1)}(\mathbf{r}, t; \mathbf{r}, t)\right] = \mathrm{Tr}\left\{\widehat{\rho}\, \widehat{E}_i^{(-)}(\mathbf{r}, t)\widehat{E}_j^{(+)}(\mathbf{r}, t)\right\}, \quad i = x, y; \quad j = x, y, \tag{1}$$

where $\widehat{E}_i^{(+)}$ and $\widehat{E}_i^{(-)}$ are the i components of the positive and of the negative frequency parts of the quantized electric field operator respectively, and $\widehat{\rho}$ represents the density operator. A quantitative measure of polarization properties of photons, at a space-time point $(\mathbf{r}, t)$, is given by the degree of polarization ([23], Eq. (31))

$$\mathscr{P}(\mathbf{r}, t) \equiv \sqrt{1 - \frac{4\ \mathrm{Det}\overleftrightarrow{G}^{(1)}(\mathbf{r}, t; \mathbf{r}, t)}{\left\{\mathrm{Tr}\overleftrightarrow{G}^{(1)}(\mathbf{r}, t; \mathbf{r}, t)\right\}^2}}, \tag{2}$$

where Det and Tr denote the determinant and the trace respectively. This quantity is always bounded by zero and unity, i.e., $0 \leq \mathscr{P}(\mathbf{r}, t) \leq 1$. It is to be noted that $\mathscr{P}(\mathbf{r}, t)$ is expressed in terms of the trace and the determinant of the matrix $\overleftrightarrow{G}^{(1)}(\mathbf{r}, t; \mathbf{r}, t)$, and hence is invariant under unitary transformations. When $\mathscr{P}(\mathbf{r}, t) = 0$, the light is completely unpolarized at the

space-time point $(\mathbf{r}, t)$ and when $\mathscr{P}(\mathbf{r}, t) = 1$, it is completely polarized at that space-time point. In intermediate cases ($0 < \mathscr{P}(\mathbf{r}, t) < 1$), the light is said to be partially polarized.

Suppose, now, that $|\psi_1\rangle$ and $|\psi_2\rangle$ represent two normalized single-photon states (eigenstates of the number operator), so that

$$\langle\psi_1|\psi_2\rangle = \langle\psi_2|\psi_1\rangle = 0, \tag{3a}$$

$$\langle\psi_1|\psi_1\rangle = \langle\psi_2|\psi_2\rangle = 1. \tag{3b}$$

We first consider a state $|\psi_{\rm ID}\rangle$ of light, which is formed by coherent superposition of the two states $|\psi_1\rangle$ and $|\psi_2\rangle$, i.e.,

$$|\psi_{\rm ID}\rangle = \alpha_1 |\psi_1\rangle + \alpha_2 |\psi_1\rangle, \quad |\alpha_1|^2 + |\alpha_2|^2 = 1, \tag{4}$$

where α_1 and α_2 are, in general, two complex numbers. In this case, a photon may be in the state $|\psi_1\rangle$ with probability $|\alpha_1|^2$, or in the state $|\psi_2\rangle$ with probability $|\alpha_2|^2$, but the two possibilities are intrinsically *indistinguishable*. The density operator $\widehat{\rho}_{\rm ID}$ will then have the form

$$\widehat{\rho}_{\rm ID} = |\alpha_1|^2 |\psi_1\rangle\langle\psi_1| + |\alpha_2|^2 |\psi_2\rangle\langle\psi_2| + \alpha_1^*\alpha_2 |\psi_2\rangle\langle\psi_1| + \alpha_2^*\alpha_1 |\psi_1\rangle\langle\psi_2|. \tag{5}$$

In the other extreme case, when the state of light is due to incoherent superposition of the two states, the density operator $\widehat{\rho}_{\rm D}$ will be given by the expression

$$\widehat{\rho}_{\rm D} = |\alpha_1|^2 |\psi_1\rangle\langle\psi_1| + |\alpha_2|^2 |\psi_2\rangle\langle\psi_2|. \tag{6}$$

Here $|\alpha_1|^2$ and $|\alpha_2|^2$ again represent the probabilities that the photon will be in state $|\psi_1\rangle$ or in state $|\psi_2\rangle$, but now the two possibilities are intrinsically *distinguishable*. Mandel [7] showed that in any intermediate case, the density operator

$$\widehat{\rho} = \rho_{11} |\psi_1\rangle\langle\psi_1| + \rho_{12} |\psi_1\rangle\langle\psi_2| + \rho_{21} |\psi_2\rangle\langle\psi_1| + \rho_{22} |\psi_2\rangle\langle\psi_2| \tag{7}$$

can be uniquely expressed in the form

$$\widehat{\rho} = \mathscr{I}\widehat{\rho}_{\mathrm{ID}} + (1-\mathscr{I})\widehat{\rho}_{\mathrm{D}}, \qquad 0 \le \mathscr{I} \le 1. \tag{8}$$

Mandel's defined $\mathscr{I}$ as the *degree of indistinguishability*. If $\mathscr{I} = 0$, the two paths are completely distinguishable, i.e., one has complete WPI; and if $\mathscr{I} = 1$, they are completely indistinguishable, i.e., one has no WPI. In the intermediate case $0 < \mathscr{I} < 1$, the two possibilities may be said to be partially distinguishable. Clearly, $\mathscr{I}$ may be considered as a measure of WPI. According to Eqs. (7) and (8), one can always express $\widehat{\rho}$ in the form

$$\widehat{\rho} = |\alpha_1|^2 |\psi_1\rangle \langle\psi_1| + |\alpha_2|^2 |\psi_2\rangle \langle\psi_2| + \mathscr{I}(\alpha_1^*\alpha_2 |\psi_2\rangle \langle\psi_1| + \alpha_2^*\alpha_1 |\psi_1\rangle \langle\psi_2|). \tag{9}$$

Clearly, the condition of "best circumstances" requires that $|\alpha_1| = |\alpha_2|$.

For the sake of simplicity, let us assume that $|\psi_1\rangle$ and $|\psi_2\rangle$ are of the form

$$|\psi_1\rangle = |1\rangle_x |0\rangle_y , \tag{10a}$$

$$|\psi_2\rangle = |0\rangle_x |1\rangle_y , \tag{10b}$$

where the two states are labeled by the same (vector) mode $\mathbf{k}$, and x, y are two mutually orthogonal directions, both perpendicular to the direction of $\mathbf{k}$ [for the sake of brevity, $\mathbf{k}$ is not displayed in Eqs. (10)]. Clearly $|\psi_1\rangle$ represents the state of a photon polarized along the x direction, and $|\psi_2\rangle$ represents that along the y direction. In the present case, one may express $\widehat{E}_i^{(+)}(\mathbf{r},t)$ in the form

$$\widehat{E}_i^{(+)}(\mathbf{r},t) = Ce^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\widehat{a}_i, \qquad (i = x, y), \tag{11}$$

where the operator $\widehat{a}_i$ represents annihilation of a photon in mode $\mathbf{k}$, polarized along the $i-$ axis, and C is a constant. From Eqs. (1), (9), and (11), one readily finds that the quantum

polarization matrix $\overleftrightarrow{G}^{(1)}(\mathbf{r},t;\mathbf{r},t)$ has the form

$$\overleftrightarrow{G}^{(1)}(\mathbf{r},t;\mathbf{r},t) = |C|^2 \begin{pmatrix} |\alpha_1|^2 & \mathscr{I}\alpha_1^*\alpha_2 \\ \mathscr{I}\alpha_1\alpha_2^* & |\alpha_2|^2 \end{pmatrix}. \tag{12}$$

From Eqs. (2) and (12) and using the fact $|\alpha_1|^2 + |\alpha_2|^2 = 1$, one finds that, in this case, the degree of polarization is given by the expression

$$\mathscr{P} = \sqrt{(|\alpha_1|^2 - |\alpha_2|^2)^2 + 4|\alpha_1|^2|\alpha_2|^2\mathscr{I}^2}. \tag{13}$$

It follows from Eq. (13) by simple calculations that

$$\mathscr{P}^2 - \mathscr{I}^2 = (1-\mathscr{I}^2)(2|\alpha_1|^2 - 1)^2. \tag{14}$$

Using the fact that $0 \leq \mathscr{I} \leq 1$, one readily finds that

$$\mathscr{P} \geq \mathscr{I}. \tag{15}$$

Thus, the degree of polarization of the out-put light in a single-photon interference experiment is always greater or equal to the degree of indistinguishability ($\mathscr{I}$) which a measure of "which-path information".

Let us now assume that the condition of "best circumstances" is achieved, i.e., one has $|\alpha_1|^2 = |\alpha_2|^2$. It then readily follows from Eq. (13) that

$$\mathscr{P} = \mathscr{I}. \tag{16}$$

This formula shows that under the "best circumstances", the degree of indistinguishability (a measure of WPI) and the degree of polarization are equal. We will now discuss the physical interpretation of this result. If one has complete "which-path information" (i.e., $\mathscr{I} = 0$), then from Eq. (16) it follows that the degree of polarization of the light emerging

from the interferometer is equal to zero. Complete "which-path information" in an single-photon interference experiment implies that a photon shows complete particle nature, and our analysis suggests that in such a case light is completely unpolarized. In the other extreme case, when one has no "which-path information", i.e., when a photon does *not* display any particle behavior, the output light will be completely polarized. Any intermediate case will produce partially polarized light. It is clear that such a phenomenon is completely quantum-mechanical and cannot be realized by the use of classical theory.

We summarize our result by saying that in a single-photon experiment the polarization properties of the light emerging from the interferometer depend on whether a photon behaves like a particle or like a wave.

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References and Notes

[1] This abbreviation is due to M. Bunge [see, for example, J.-M. Lévy-Leblond, Physica B **151**, 314 (1988), and also B.-G. Englert, Phys. Rev. Lett. **77**, 2154 (1996)].

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